Winchester Math Curriculum Grade 4

Subject	Mathematics
Grade/Course	Grade 4
Unit of Study	Unit 1: Multiplicative Thinking
Pacing	August / September
Unit Summary	Unit 1 begins the year with a study of multiplication and division, focusing in particular on models, strategies and multiplicative comparisons. Students solve multiplication and division story problems and investigate factors, multiples, prime and composite numbers. Students then extend the idea of multiplicative comparison into the arena of measurement,by developing a deeper understanding of the metric units for length, mass, and liquid volume.

Overarching Mathematical Practices

4.MP.1   Make sense and persevere in solving problems.
4.MP.2   Reason abstractly and quantitatively.
4.MP.3   Construct viable arguments and critique the reasoning of others.
4.MP.4   Model with mathematics.
4.MP.5   Use appropriate tools strategically.
4.MP.6   Attend to precision.
4.MP.7   Look for and make use of structure.
4.MP.8   Look for and express regularity in repeated reasoning.

Unit CT Core Content Standards

4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.1

4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

4.OA.B.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

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4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

"Unwrapped" Standards	
Skills	**Content**
Interpret	- Multiplication equations as a comparison - Remainders
Represent	- Multiplicative comparisons as multiplication equations - Word problems using equations with a letter for the unknown quantity - Measurement quantities using diagrams
Solve	- Word problems involving multiplicative comparison (multiply or divide) - Multi-step word problems with whole numbers in all four operations - Word problems involving distances, intervals of time, liquid volumes, masses of objects, and money (using four operations)
Assess	Reasonableness of answers
Find	- All factor pairs for whole numbers (1-100) - Whole number quotients and remainders with up to 4-digit dividends and 1-digit divisors
Recognize	A whole numbers is a multiple of each of its factors
Determine	- If a given whole number is a multiple of a given 1-digit number (1-100) - Prime or composite (1-100)
Multiply	- Whole numbers up to 4-digits by a 1-digit whole number - Two 2-digit whole numbers
Illustrate and explain	Calculations by using equations, rectangular

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	arrays, and/or area models
Know	● Relative sizes of measurement units within one system of units
Express	● Measurements in a larger unit in terms of a smaller unit
Record	● Measurement equivalents in a two-column table

Essential Questions	Corresponding Big Ideas
1. What strategies can we use to learn to multiply and divide? 2. How do factors and multiples promote fact fluency and automatic recall? 3. How do we become good problem solvers?	1. We can use open number lines, arrays, ratio tables, and equations to solve multiplication and division problems. 2. Understanding the relationship and patterns between factors and multiples develops understanding of our number system that promotes efficient, appropriate, and flexible single digit calculations. 3. We find our own entry points to the question, consider the meaning of a problem, and look for appropriate and efficient ways to solve it.

Evidence of Learning - Assessment

Pre/Post Assessment	Interim Assessment	Additional Evidence of Learning
● Unit 1 Pre-Assessment and Student Reflection - Module 1, Session 3 ● Unit 1 Post-Assessment and Student Reflection -Module 3, Session 5 ● Number Corner Baseline Assessment	● Multiplication and Division Checkpoint - Module 3, Session 2	Options ● Exit tickets ● Observational Assessments ● Cover Up - M2, S3 ● Arrays to One Hundred - M2, S4 ● The Multiple Wheel - M2, S5 ● Spinning Around Multiplication - M2, S6 ● Products Four in a Row - M3, S1 ● Dragon's Gold - M3, S4 ● Math Practices Observation Chart

Smarter Balanced Interim Assessment

Smarter Balanced General Scoring Rubrics - 4 Rubrics included - Score Pt 4 to Score Pt 1
· **Smarter Balanced Interim Blocks**

· Interim assessment blocks may be used for a variety of assessment purposes, including: pre/post, interim and formative (additional evidence of learning).

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· The Style Guide, which aligns with the expectations of Smarter Balanced Assessments, will support the creation of unit- and standard-aligned items for instructional use.

· The items on the interim assessments are developed under the same conditions, protocols, and review procedures as those used in the summative assessments. Therefore, they assess the same Common Core State Standards, adhere to the same principles of Universal Design in order to be accessible to all students, and provide evidence to support Smarter Balanced claims in mathematics and ELA/literacy. The interim assessment items are non-secure but non-public. This means that educators may view the items, however, they should not be made public outside of classroom, school or district.

· **Unit-aligned Smarter Balanced Interim Assessment Block (IAB)*:**

Interim Assessment Block - access through CSDE Assessment Portal
● IAB - Operations and Algebraic Thinking

Learning Plan

Researched-based Instructional Resources and Methods

Sequence of Instruction:
Number Corner → Problem + Investigations →Work Places → Math Forum*→Daily Practice or Home Connection

Bridges Number Corner - The focus areas of Number Corner Aligned to Unit 1 are:
● Multiplication/Division
  ○ Write equations to represent a multiplicative comparison
  ○ Solve story problems
  ○ Find all factor pairs for a whole number
  ○ Understand that a whole number is a multiple of each of its factors
  ○ Multiply two-digit by 1-digit and 2-digit by 2-digit whole numbers
  ○ Write simple expressions to record calculations with numbers
● Fractions
  ○ Recognize equivalent fractions
  ○ Explain addition of fractions
  ○ Solve story problems adding fractions
  ○ Multiply fractions by whole numbers
● Measurement
  ○ Express larger units in terms of smaller units
  ○ Record equivalent measurements
  ○ Solve story problems involving distance using addition/multiplication of fractions
● Patterns
  ○ Number of zeros in product when multiplying by powers of 10.
  ○ Generate a number or shape pattern that follows a given rule.
● Place Value
  ○ Read and write multi-digit whole numbers with numerals, word form, and expanded form
  ○ Demonstrate an understanding that in a multi-digit number each number represents ten times what it represents in the place to its right

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Bridges - Whole Group, Small Group and Independent Problem Centered Activities

Module 1	Module 2	Module 3	Module 4

Problem + Investigation
  ● Sessions 1,4,5
Problem String
  ● Sessions 2-3
Work Place
  ● None
Math Forum
  ● Session 6
Daily Practice
  ● Sessions 1-6
Home Connection
  ● Sessions 2,4,6
	Problem + Investigation
	  ● Sessions 1-3, 5-6
	Problem String
	  ● Session 10
	Work Place
	  ● Sessions 3-6
	Math Forum
	  ● Session 2
	Daily Practice
	  ● Sessions 1-6
	Home Connection
	  ● Sessions 2,4,6
		Problem + Investigation
		  ● Sessions 2,3
		Problem String
		  ● None
		Work Place
		  ● Session 1,2,4,5
		Math Forum
		  ● None
		Daily Practice
		  ● Sessions 1-5
		Home Connection
		  ● Sessions 2 + 4
			Problem + Investigation
			  ● Sessions 1-3
			Problem String
			  ● None
			Work Place
			  ● None
			Math Forum
			  ● None
			Daily Practice
			  ● Sessions 1-3
			Home Connection
			  ● Sessions 1 + 3

Possible Misconceptions	Teacher Moves

1. Students may struggle when applying their knowledge of multiplication and division facts to multiplicative situations since all of their previous experience was with equal groups and array models.
2. Students may confuse additive and multiplicative situations.
 	
3. Students who struggle determining what operation to use when solving a problem.
	1. They need many experiences connecting facts to the language of multiplicative comparisons. Using concrete models will support students in making this connection.
	
	2. They need a variety of problems to model and discuss. Identifying what they know from the information in the problem and focusing on the question will help them to make sense of the problem. They should also consistently ask themselves if their answer makes sense.
	3. These students need additional experiences understanding operations in a variety of situations (See situation types in links under Math Teaching Practices). They should have explicit practice with various problem solving strategies, including:
	  a. restating the problem in their own words.
	  b. Identifying given, needed, and wanted information.
	  c. make a model or drawing a picture.
	  d. making a list.
	  e. acting it out.
	  f. finding a pattern.
	  g. writing an equation,
	  h. revisiting the question and asking

	themselves if the solution makes sense.
4. Students who do not have conceptual understanding or have focused work on division procedures tend to write answers to problems using "r" notation for remainders. For example, 16 students are going canoeing. If each canoe hold 3 canoers, how many canoes wil1 they need? The answer 5 r 1 makes no sense in this situation.	4. Students need to focus on the question and reasonableness of solutions using strategies such as models, pictures, and acting it out.
5. Student often confuse the terms factor and multiple.	5. Emphasizing the term factor as one of the numbers multiplied to get a product throughout all the work with multiplication, and expecting students to use the term as they discuss math situations. Telling student that you multiply to get a multiple or defining multiples of a number as the products of the number is helpful. The more experience students have with these terms, the more accurate they will become when using them.
6. When listing multiples of a number, students may forget to include the number itself.	6. Reminding students that multiples are the products of number leads to a discussion of why a number is a factor and a multiple itself, which is a result of the identity element of multiplication (a x 1 = a)
7. Student may become confused about whether 1 is a prime or composite number, when actually it is neither prime nor composite because it only has one factor itself.	7. Developing precise definitions should help to eliminate this misconception.

Vocabulary and Representations

Tier 2 (Academic Vocabulary)	Tier 3 (Domain Specific Vocabulary
Calculation	Additive Comparison
Capacity	Area Model* of Multiplication
Comparison	Array* (Rectangular, Tile, Hidden)
Cup	Associative Property of Multiplication
Data	Centimeter*
Dimension	Commutative Property of Multiplication
Distance	Comparison Statement
Divide/Division*	Composite Number
Efficient	Computational Fluency
Equivalents	Customary System
Estimation*	Decimals*
Express	Dividends
Factor*	Divisor
Gallon	Equation*

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Gram*	Fraction (unit fraction)*
Horizontal	Kilogram*
Illustrate	Line Plot*
Interpret	Linear Measurement
interval*	Liquid Volume*
Intervals of time*	Factor Pairs*
Liter*	Measurement Scales
Mass*	Measurement Unit
Meter*	Mental Computation
Model	Metric System*
Multiple*	Milliliter*
Multi-step	Multiplication Table
Ounce	Multiplicative Comparison
Pint	Multiply/Multiplication*
Pound	Number Line* (Open)
Predict	Prime Number
Product*	Properties of Operations
Quart	Quotient*
Reasonableness	Ratio Table
Relative size	Systems of Units (ex. km, m, cm)
Remainder*	
Rounding*	
times as many*	*Smarter Balanced Vocabulary is focused on major mathematical concepts. (Not all possible words have been identified by SBAC)
times as much*	
Unit	+ Students are not responsible for these vocabulary words at this grade level, however they should have some understanding of the mathematical concept.
Vertical	

Mathematics Teaching Practice Resources

1. **Bridges** - Reference Math Practices in Action Notes - The notes identify how particular mathematical practice is employed in a specific activity.
2. Illustrating the Standards for Mathematical Practice
3. Math Practices Question Stems
4. Illustrative Math – Grade 4 - Resources and activities for grade 2 aligned by standard.
5. Journal Prompts for Math
6. Bridges Interactive Math Manipulatives
7. Bridging Practices - UCONN – Training and a Task Repository to develop and support student capacity for argumentation in mathematics
8. Teacher/Student Actions
9. Accountable Talk Moves
10. Contribution Checklist
11. Sentence Frames that Can Build Metacognitive Thinking
12. Sample Language Frames for Mathematics
13. Building a Mathematical Mindset Community

14. [Math Practice Standards Posters](#) Gr. 4 -5
15. **[Implementing the Standards of Mathematics Practice](#)**
16. Bridges [Interactive Math Manipulatives](#)
17. [Multiplication and Division Situations](#)
18. [Addition and Subtraction Situations](#)
19. [Beginning to Problem Solve with I Notice, I Wonder](#)
20. [K - 5 Math Teaching Resources](#)
    - [Multiplication as Comparison](#)
    - [Word Problems - Multiplicative Comparison](#)
    - [Word Problems - Interpreting Remainders](#)
    - [A Remainder of One](#)
    - [Exploring Multiples](#)
    - [Prime or Composite](#)
21. LearnZillion
    - [Compare Numbers Using Bar Models](#)
    - [See Multiplication as a Comparison using Number Sentences](#)
    - [The Commutative Property](#)
    - [Solve Multiplicative Comparison Word Problems](#)
    - [Find All the Factor Pairs of a Number](#)
    - [Determine if a Number is Prime or Composite Using Area Models](#)

Suggestions for Differentiation, Scaffolding and Intervention

Differentiation or Intervention
Any teacher moves/strategies that address misconceptions can be used in differentiation or as interventions.
Math Teaching Practice Resources contain resources that provide opportunities for differentiation,
intervention, or extension aligned to the strategies below.

- [How to Select Math Intervention Content](#)
- [Coherence Map in Math](#) – The coherence map shows how standards within and across grades build upon each other. You can use the map to assist you in to build student understanding by linking together concepts within and across grades and identify gaps in a student's knowledge by tracing a standard back through its logical prerequisites.
- [CT Dept. of Education Evidence-based Practice Guides](#) – These guides provide links to "evidence-based activities, strategies and interventions (collectively referred to as 'interventions')."
- Evidenced-based strategies for supporting struggling students (U.S. Dept. of Education – [What Works Clearinghouse)](#)
- Ensure instructional materials are systematic and explicit. In particular, they should include numerous clear models of easy and difficult problems, with accompanying teacher think alouds.
- Provide students with opportunities to solve problems in a group and communicate problem-solving strategies.
- Teach students about the structures of various problem types, how to categorize problems based on structure, and how to determine appropriate solutions for each problem type.
- Students should work with visual representations of mathematical ideas.
- If visual representations are not sufficient for developing accurate abstract thought and answers, use concrete manipulative first. (Include the next line for middle school and older students only) Although this can also be done with students in upper elementary and middle school grades, use of manipulatives with older students should be expeditious because the goal is to move toward understanding of and facility with visual representations and finally to the abstract.

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- Provide carefully constructed questions to help direct students in determining what to do to solve problems, but they shouldn't be told how to reach the solution.
- Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

Intervention for facts
- Provide about 10 minutes per session of instruction to build quick retrieval of basic arithmetic facts. Consider using technology, flash cards, and other materials for extensive practice to facilitate automatic retrieval.
- For students in K -2 explicitly teach strategies for efficient counting to improve the retrieval of mathematics facts.
- Teach students in grade 2-8 how to use their knowledge of properties, such as commutative, associative, and distributive to derive facts in their heads.
- How to Promote Acquisition of Math Facts – Intervention for struggling students
- National Center on Intensive Intervention - Basic Facts
- Once a strategy has been taught, it is important to reinforce it. The reinforcement or practice exercises should be varied in type and focus as much on the discussion of how students obtained their answers as on the answers themselves.
- Having students work in groups (as opposed to handing your bright students a workbook to work on when the classroom material isn't challenging enough) with other children ready for advanced material shows them that mathematics is not a solitary discipline -- mathematics is exciting and vibrant and creative and fun.
- Concrete, Representational, Abstract Progression

EL Strategies
- Colorin Colorado – A Bilingual site for educators and families of English learners
- Stanford University - Principles for Mathematics Instruction of ELs
- CT State Dept. Of Education English Learner Standards and Resources
- Nonverbal responses, such as thumbs up, will help you check for understanding without requiring students to produce language. ELLs can participate and show that they understand a concept, or agree or disagree with an idea, without having to talk. This is especially important for students whose comprehension of English is more advanced than their ability to speak the language.
- Pre-teach vocabulary in ways that connect to students' prior knowledge.
- Display posters of graphic representations of vocabulary words.
- http://www.cal.org/siop/lesson-plans/
- Provide support to assist in explaining thinking with sentence starters and work banks.
- Use Work Place Sentence Frames or other sentence frames to assist students in math discourse.
- Speak slowly and use clear articulation. Reduce the amount of teacher talk and use a variety of words for the same idea. Exaggerate intonation and place more stress on important new concepts or questions. After asking a question, wait for a few moments before calling on a volunteer. Writing the question on the board will also help.
- English language learners are not always able to answer the questions posed to them, especially when the questions are open-ended. Provide support for and improve the participation of students with lower levels of English proficiency by using a prompt that requires a physical response, like "Show me a half, a third, etc.." or "Touch the larger number."
- Increase academic language knowledge for English learner success.

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Extension:
- Students can create their own multiplication and division problems
- Students can create story problems that have a two step process.
- Give the student the product and ask them to find out how many different factors could be used to reach the given product. Explain the reasoning and strategy used.
- Once students have solved several set of problems, those who easily identify the information in a problem and make connections between previous work with multiplication and the work with division can begin to solve problems that involve remainders.
- Ask students to create and solve their own story problems

Interdisciplinary Connections

Children's Literature *** Bridges recommended titles - # Titles embedded in Bridges Units**

*Jim and the Beanstalk by Raymond Briggs
*You Can Count on Monsters by Richard Evan Schwartz
*The Great Divide: A Mathematics Marathon by Dayle Ann Dodds
*Divide and Ride by Stuart J. Murphy

A Remainder of One by Elinor J. Princzes
*A Grain of Rice by Helena Clare Pittman

ELA
CCSS.ELA-LITERACY.SL.4.1

Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 4 topics and texts*, building on others' ideas and expressing their own clearly.
CCSS.ELA-LITERACY.SL.4.1.A

Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion.
CCSS.ELA-LITERACY.SL.4.1.B

Follow agreed-upon rules for discussions and carry out assigned roles.
CCSS.ELA-LITERACY.SL.4.1.C

Pose and respond to specific questions to clarify or follow up on information, and make comments that contribute to the discussion and link to the remarks of others.
CCSS.ELA-LITERACY.SL.4.1.D

Review the key ideas expressed and explain their own ideas and understanding in light of the discussion